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FULL TENSOR REPRESENTATION OF ANISOTROPY IN HYDRAULIC CONDUCTIVITY: EFFECTS OF SIMULATING DISCHARGE OF GROUNDWATER TO LAKES

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Summary. *It is well known that anisotropy plays an important role in discharge of groundwater to lakes; groundwater flows almost horizontally to a lake then over a short distance changes direction to an almost vertical direction in the vicinity of or right below the lake. At this scale anisotropy becomes important. However, usually most simulation studies assume that the principal directions of the hydraulic conductivity tensor are horizontal and not aligned with the sloping lake beds and maybe not even with sloping sedimentary layers next to the lake. FEFLOW was used to study the effects of including a full tensor representation of the individual layers (sediments, lake bed) on flow patterns and discharge to characteristic lakes in Denmark. Sloping layers with different angles to the horizontal have been incorporated into a conceptual 2D cross-sectional model. Differences are found between simulations, where the principal directions follows the horizontal axis and simulations, where the principal directions are aligned with the sloping sediment layers and lake bed; (1) a more complicated flow pattern beneath the lake, sometimes with flow reversals towards land and (2) thus, also a different discharge pattern to the lake deviating from the often assumed classical exponential distribution. The results are sensitive to the thickness of the layers/lake beds, anisotropy, and slope (angle of principal direction) but with the biggest effect seen in the distribution of discharge to the lake, e.g. where 90% of the discharge now occurs of an ~ 50 m wider lake shore. The effect on the total discharge and near-shore discharge are minor. Currently we are conducting geophysical off-shore to on-land investigations to reveal lake bed and layer sedimentation (and angles) and the model will be used to simulate discharge to these lakes and compare with observations.*

1 INTRODUCTION

Natural hydrogeological systems are usually anisotropic in hydraulic conductivity. The deposition of geological structures and the presence of differently sized particles, like clays, within these structures and with a clear difference in one of its dimensions can explain anisotropy. Often anisotropy factors with the horizontal hydraulic conductivity (K_h) about 10-100 times higher than the vertical hydraulic conductivity (K_v) are observed. Models usually considers this situation by including a constant anisotropy factor (or a zone-based ratio), where the principal directions of anisotropy typically are aligned with the coordinate system defining the computational mesh.

Groundwater flow changes direction when an aquifer interacts with a lake, river, or any other surface water body. For example, the flow pattern changes from an essentially horizontal component to close to an essentially vertical component when groundwater discharges to a lake. Such a flow system is especially sensible to the degree of anisotropy[1].

Nevertheless there is another condition that, most of the times, is not considered, i.e., the presence of tilted sediments below the lake bed. These sediments can have a high anisotropy ratio as well, but the angle of anisotropy can be different from the rest of the aquifer. The origin of this is the modification of the lake water level associated with changes in e.g. the climate, catchment size, anthropogenic impacts such as dams, weirs, or simply the use of lake water that all change the sedimentary conditions. The velocity of sedimentation near shore in lakes can be very fast and the possibility of finding sediment layers below the lake bed with different angles relative to the horizontal is not atypical, this is what we define as a lake bed layer. Thus, a full tensor representation is needed to simulate flow in systems with varying degrees of angles of anisotropy.

The aim of the study is to investigate the effects of tilted sediments with different angles of anisotropy on groundwater-lake interactions; specifically to study how these affect discharge to a lake in terms of amount and distribution.

2 METHODOLOGY

A hypothetical 2D cross-sectional model that simulates groundwater flow from a shallow aquifer to a lake was developed based on the software FEFLOW. The main advantage of FEFLOW is the possibility of modifying the angle of anisotropy in different areas of the modeled section. A series of simulations were conducted representing isotropic/anisotropic conditions (K_h/K_v=1, 10, and 50) with different angles (1^o, 3^o, 5^o and 10^o) of sediments layers (and principal axes) below the lake bed, and for variable thickness of the sediments (1 m, 3 m, and 6 m). The results were analyzed based on the effect on the total discharge to the lake, the modification of the discharge pattern from the aquifer to the lake (that usually follows an exponential decrease from the shore line[2]) and the variations of the flow path below the lake, and the distance over which 30%, 50% or 90% of the total discharge reaches the lake.

3 MODEL CONSTRUCTION

The model is a 2D section of 1000 m length, 28-33 meters thickness with a discretization of 418852 elements and 211554 nodes having triangular elements with approximately 50 cm sides. The variable thickness is because of a continuous slope from land to the deeper parts of the lake. The flow simulations are steady state. The K_h is $7.9.10^{-4}$ m/s for both the aquifer and lake bed layers (a common value for fluvial and glacial sand). The model setup reflects conditions at a Danish lake[3]. The boundary conditions are; (1) Constant head at the land side vertical boundary and along half of the upper part of the model representing the lake resulting in a constant gradient (0.0006) in all simulations, (2) no-flux along the top land-side boundary and lake-side vertical boundary, and (3) impermeable bottom.

The layers that are located under the lake in Figure 1 represent the areas where the hydraulic properties have been modified. The thickness of the layer is constant along all the length but variable for each case study and parallel to the upper border of the model (or lake bed). The inclination of these layers means that the angle of anisotropy varies from the horizontal to the direction of maximum hydraulic conductivity measured counter-clockwise. For example, taking as a starting point the lake border, the line of maximum hydraulic conductivity would be a line with an inclination of 1^o, 3^o, 5^o or 10^o (depending on the scenario considered) with respect to the horizontal and it will continue in this direction until reaching 1, 3 or 6 meters deep where the direction will change to horizontal. The rest of the aquifer has the same anisotropy ratio, but the inclination is 0. FEFLOW includes these effects by computing the full hydraulic conductivity tensor with off-diagonal terms.

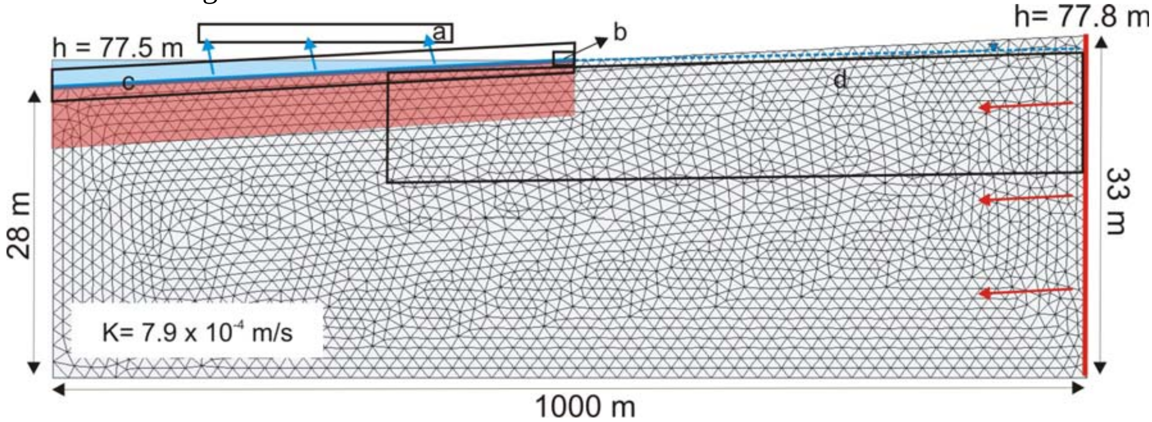

Figure 1. Model domain and specified head boundary conditions, all other boundary conditions being no-flux (the discretization is not as refined as in the model, but the shape of the elements is analogous). The effects of anisotropy were investigated at the following scales: a) Total discharge across the whole lake bed, b) Discharge into the lake at the lake shore, c) Distance of 30/50/90% discharge, d) Flow paths

4 RESULTS

The analysis compares the different anisotropy simulations with each other and with that having no sediment layer below the lake bed. We have applied different scales for the research depending on the objectives (Fig. 1).

4.1 Total discharge

The initial question was if the total discharge to the lake was affected. The differences in percentage (negative implying less discharge than the case without a layer) are shown in Tables 1 and 2. The differences never exceed -0.4 % for the most extreme situation (6 m thick layer and 10o of inclination) when the anisotropy ratio was 10 (Table 1). A higher anisotropy ratio of 50 gave larger differences (Table 2), but still very low (the maximum reached was -0.83).

In general the effect of this layer is a very small decrease of the amount of water discharging to the lake. This is related to the greater difficulty for water to flow through sediments with lower permeability and the angle of inclination in most of the cases results in a higher resistance to flow into the lake. But there are exceptions when the lake bed layer is thinner and the angle is big. In this case it is possible to detect a slight increase (positive) in total discharge.

Thickness\Angle	1	3	5	10
1 m	-0.002 %	-0.013 %	-0.015 %	0.028 %
3 m	-0.034 %	-0.094 %	-0.113 %	-0.094 %
6 m	-0.083 %	-0.198 %	-0.303 %	-0.374 %

Table 1. Difference in discharge to lake comparing a model with homogenous anisotropy with a model including a lake bed layer (anisotropy ratio of 10)

Thickness\Angle	1	3	5	10
1 m	-0.037 %	-0.024 %	0.092 %	0.486 %
3 m	-0.177 %	-0.311 %	-0.192 %	0.516 %
6 m	-0.370 %	-0.746 %	-0.826 %	-0.059 %

Table 2. Difference in discharge to lake comparing a model with homogenous anisotropy with a model including a lake bed layer (anisotropy ratio of 50)

4.2 Discharge into the lake

The discharge pattern from aquifers to lakes has been studied by different authors [1,2,3] that indicated the existence of an exponential decrease of discharge from the lake border to more distal positions. Figure 2 shows the simulated discharge at the lake border for variable conditions of anisotropy. The main effect of anisotropy is a sharper decrease of the discharge in the areas closer to the border.

The presence of the layer produces a slightly earlier discharge into the lake compared with the fully anisotropic case. The inclination of the sediments is the most sensitive parameters; when the angle is higher, the discharge is higher in the proximal areas to the lake border. The discharge in the areas closest to the lake border can increase by 50 % compared to the homogeneous case. For an angle of 1o the effect is almost negligible independently of the thickness of the layer. As a reference, the change in discharge in the first meters of the lake is almost the same considering a layer of 1 m thickness with inclination of 10^o and a layer of 6 m thickness with an inclination of 5^o. A change in discharge occurs in the first meters, but, as the total discharge is little affected this change can be considered as a local effect.

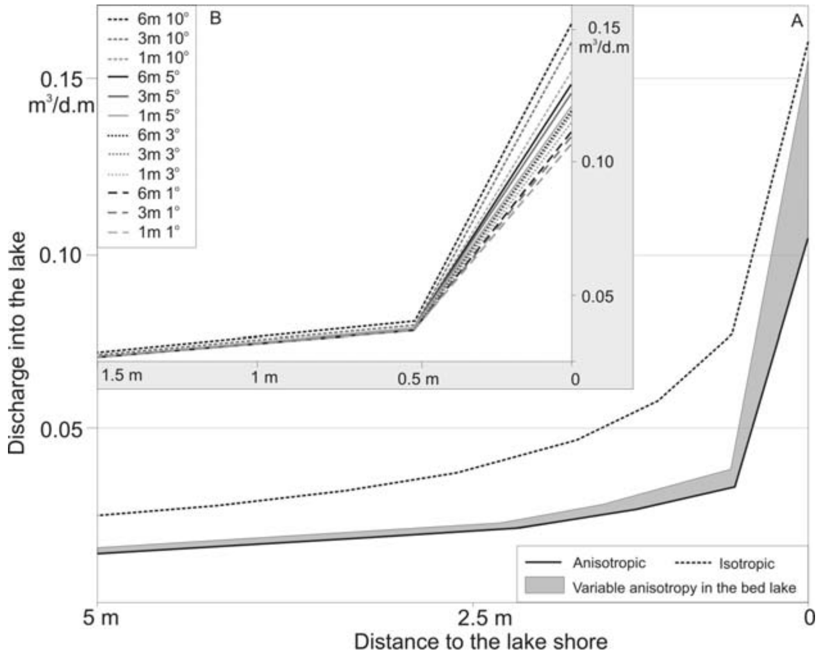

Figure 2. A. Discharge to the lake vs distance from lake border for isotropic conditions, homogeneous anisotropy (K_h/K_v= 50) and variable anisotropic hydraulic conductivity characteristics and lake bed thicknesses. B. Details of the lake bed layer effects. The decrease in the discharge associated mainly with the inclination of the sediments is demostrated with the order of the lines (from maximum to minimum) plotted in the graph.

4.3 Distance of discharge

Another metric was evaluated measuring the changes in the distance over which 30 %, 50 % and 90 % of the total of the discharge occur. Two main scenarios were considered; K_h/K_v = 10 and 50.

Homogeneous			1°	3°	5°	10°
6.79 m		30 %	6.78	6.78	6.3	5.81
20.61 m	**1m**	50 %	20.83	20.83	20.35	19.86
112.4 m		90 %	112.4	111.92	111.43	110.47
		30 %	6.78	6.3	5.81	4.85
	3m	50 %	20.83	20.35	19.38	17.93
		90 %	111.92	110.95	109.98	107.08
		30 %	6.78	6.3	5.03	4.36
	6m	50 %	20.83	19.86	18.9	16.47
		90 %	111.92	110.47	109.01	103.2

Table 3. Distances where 30, 50 and 90 % of the discharge is transferred to the lake in the different scenarios. Same values for the homogenous anisotropic case are also given.

The case of Kh/Kv = 10 is presented in Table 3. The main change associated with the introduction of the layer with a variable anisotropy inclination is the decrease of the distance when the angle or the thickness of the layer was increased. The changes in the most extreme cases showed a movement of less than 10 meters over a total distance of more than 100 m, or that the same amount of discharge is occurs over 10 % less distance. In most of the scenarios the changes were lower than 1-2 meters. The angle of 1o almost did not change the results, especially for the distances, where 30-50 % of the discharge reached the lake.

The results for a higher anisotropy ratio (K_h/K_v= 50) show much larger differences, Figure 3, ranging from values lower than 1 m to maximum values of almost 50 m. The distance for the lake bed layer is 201.07 m and without it is 247.58 m; in this case the reduction of the distance where discharge is taking place decreases almost 20 %. The thickness of the layer has a higher impact than layer inclination. So even though there are only small changes in discharge near the lake shore (see previous section), the 90% discharge, for example, is greatly different depending on the thickness and inclination of the sediment layer.

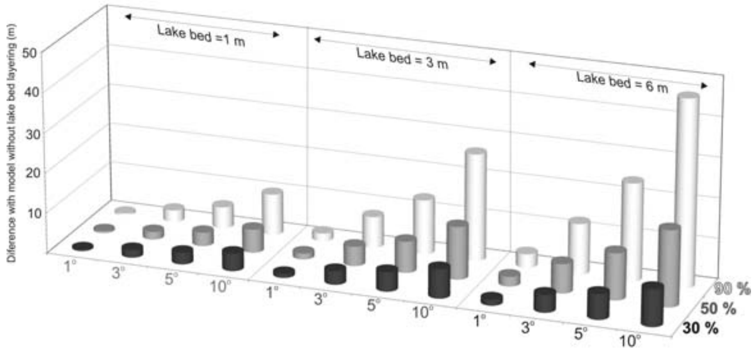

Figure 3. Differences in the distance where 30, 50 and 90 % of the discharge has reached the lake compared to a system without the lake bed layer. The distances for the reference model (without the layer) are: 247.58 m (90 %), 47.00 (50 %) and 15.50 m (30 %)

4.3 Flow paths

The thickness of the layer below the lake bed and its inclination also affects the path that groundwater is following from the aquifer to the lake. Here a qualitative comparison of the different scenarios is presented by selecting a few of these and indicating the paths that a particle located at variable depths in the aquifer at the land side (1, 8, 18 and 28 meters from the bottom of the aquifer in the right side of the model) will take.

The flow paths for the two anisotropy ratios considered in this work (10 and 50) are presented. The main difference is the increased distance over which groundwater discharges to the lake when increasing the anisotropy ratio (Figure 4). The variation is largest for the flow paths that are deeper. In the case of the K_h/K_v= 50 the deepest flow path almost reaches the boundary of the model. This forces flow upward because of the no-flow boundary at the left side of the model.

The changes in the lake bed thickness and inclination do not affect the flow paths all the way except where the properties change; that is, the last few meters before discharging to the lake. A reversal of flow is observed in the area close to the lake bottom. The start of this process is determined by the thickness of the layer and the intensity (the inclination of the flow path backwards) by the inclination of the layer. The anisotropy factor greatly amplifies flow reversal from almost negligible for thin layers with a low angle to easily detectable changes in the flow paths (Fig. 4).

It is possible to quantify the “flow reversal effect” by comparing the differences in the distance from the point that the flow path would reach with and without the lake bed layer. For the lower anisotropy factor the changes increase from 0 m to less than 5 m in the proximity of the lake border, to 10 m in the middle of the lake, reaching a maximum difference of 20 m at the more distal points from the lake border. If the anisotropy factor is higher, then in areas close to the lake border the differences are almost negligible, but further off-shore the “flow reversal effect” increase to 15 m, 30 m and 20 m. In this case the maximum difference is reached in the middle of the lake instead of at the off-shore end, different than in the previous case studied. This can be a boundary effect due to the no-flow limit of the left lateral border of the model.

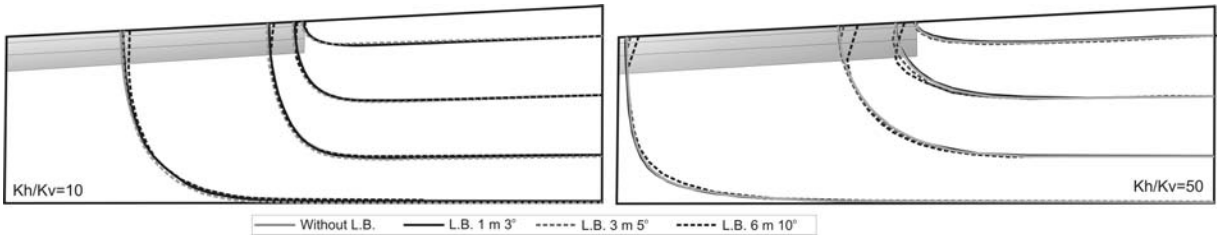

Figure 4. Flow paths for K_h/K_v= 10 and K_h/K_v= 50 (L.B. – Lake bed)

5 DISCUSSION

The total discharge to the lake was almost not affected by the lake bed layer. This is related to the very slight modification of the boundary conditions that are the same and only the inclusion of a lake bed layer. Here it has to be re-called that the lake bed layer has the same K_h as the rest of the aquifer and with a fixed-gradient system the overall discharge is little affected by the inclination and thickness of a lake bed layer. Also, the boundary conditions, with only one area of input and one area of output, determine the total discharge to the lake. A modification of the left border of the model to an open boundary condition could change this.

The increase of the discharge from the groundwater to the lake in the proximity of the lake border can be associated with the resistance to flow that is created by the tilted layers. Groundwater flowing from the inland reaches these layers with different properties (like a barrier) forcing the upwards movement of flow. Another possibility is that the flow reversal contributes to increase the discharge in the first meters of the lake.

The biggest effect of an inclined layer is seen in the distribution of flow along the whole lake border; it is shown that 90% of the total discharge can be distributed over 10-50 m less over the lake bed than in the case without a layer. In this case the thickness of the lake bed layer plays seems more relevant. The reason is that in this case we are considering a more general process, i.e., the effects are higher when we introduce a scenario with more anisotropy that is when the bed lake layer is thicker.

The simulations show the presence of "flow reversals". These effects are rather small, but in the case of high anisotropy do show that when groundwater reaches the lake bed layers tend to move slightly against the overall head gradient because of the angle of anisotropy With the introduction of the inclination of the anisotropy, the preferential flow path would be (in the case of the 6 m thickness and 10^o angle), from 6 meters below the top of the section, a line with 10^o inclination moving upward towards the lake border. Nevertheless this direction is against the full flow system (that is moving in the opposite direction) so that the results obtained are a combination of both effects. The groundwater when in reaching this area is modifying the flow track that was following and moves backwards but not in the line of preferential flow.

6 CONCLUSIONS

The effect of an inclined layer with different anisotropy angles with respect to the rest of the system was simulated using FEFLOW. The total discharge was little affected by the introduction of a layer (always lower than 1 % of change. The discharge pattern near the lake border increased due to the presence of this layer likely caused by the hydraulic contrast between this layer and the aquifer. The off-shore distance of discharge into the lake is increased with a layer and in this case the thickness of the layer is most sensitive to the results. The distance where 90 % of groundwater has discharged into the lake could change up to 46 m with a layer of 6 m thickness and 10^o of inclination (and assuming an anisotropy ratio of 50). This represents an interesting observation as it could affect the location of contamination inputs to lakes. The reversal in the flow paths is also a situation that can affects more detailed studies of the interaction between lakes and groundwater as these flow paths can introduce an error for groundwater sampling or in the use of tracers. The theoretical study presented in this work represents the background necessary for the next step in application to natural conditions. The objective now is to extend this research to the conditions at a lake in Denmark. This study has neglected any heterogeneity and it is evident that this likely will play an even greater role. It is the plan to study this first by introducing a contrast in K_h between the lake bed layer and aquifer.

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