NUMERICAL ANALYSES OF NONLINEAR (SEMI)DISCRETE RICHARDS EQUATION FOR FINITE DIFFERENCE SCHEMES: EFFECTS OF CAPILLARY PARAMETERS.

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We consider a nonlinear PDE governing water flow in unsaturated porous media, based on mass conservation and on Darcy's law. The latter expresses a quasi-linear flux-gradient law with nonlinear pressure-dependent permeability K(h). For transient flow, the water content □(h) is another nonlinear variable of the problem. The resulting PDE is cast in a conservative, mixed variable formulation.
In the 1rst part of the paper, we analyze stability/convergence properties of the (linearized) finite difference scheme under various assumptions (degree of implicitness, explicit linearization scheme, special treatment of the gravitational term). The results suggest criteria concerning time step and mesh size. However, such linearized analyzis is not sufficient to fully capture the behavior of Richards' equation.
In the 2nd part, we focus on the numerical analyzis of nonlinear convergence of the iterative scheme (fixed point / modified Picard). We look at the time semi-discretized PDE, i.e., without discretizing the spatial operator (for a consistant scheme, this can be viewed as the limit of the fully discrete scheme scheme as □x□0). The semi-discrete scheme is analyzed in detail, especially for the steady state case, where a single infinite time-step is introduced in the transient nonlinear scheme. In this case, the nonlinear convergence properties are entirely dominated by the nature of the Picard iteration scheme and by the nature of the spatial operator of the Richards equation. Our analyses focus especially on the effects of a dimensionless capillary length scale ratio related to (i) the geometry of the problem (water table depth) and (ii) the exponent parameter in the K(h) relationship (unsaturated conductivity-pressure curve). Similar analyses are being developed for the full transient case, and for the complete Darcy-Muskat equations of 2-phase flow (e.g. air-water flow).
The numerical analyses are backed-up by numerical experiments using an implicit finite volume code (BIGFLOW 3D). The code is executed under a layer of PYTHON and MATLAB commands, in order to facilitate the launching of multiple simulations and the post-processing of results. For each time step, an outer loop implements the modified Picard iterations for the nonlinear algebraic system, and an inner loop implements the Preconditioned Conjugate Gradients for the matrix solver.