A NOVEL PROCEDURE FOR THE SOLUTION OF THE HETEROGENEOUS ANISOTROPIC TRANSPORT PROBLEM. PART 2: THE TIME-DEPENDENT FLOW AND CONVECTIONDIFFUSION PROBLEM

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Anisotropic problems arise in various areas of science and engineering, for example groundwater flow simulations, pretroleum reservoir simulations, transport problems, … The present work is articulated in two companions papers: in the present one, a numerical solution of the 2D time-dependent flow and convection-diffusion problem in strongly heterogeneous media is proposed. Coefficients of tensor diffusion in the transport problem are proportional of the velocity field components and the tensor is symmetric and positive definite. The unknowns of the problem are the potential H of the flow field and the transported variable u and the governing PDEs are discretized using generally unstructured triangular Delaunay meshes. The flow problem is solved over the physical triangular mesh, according to a Lumping Mixed Hybrid Finite Element in triangle-centred finite volume formulation, which guarantees the continuity of the velocity field across triangle sides. The transport problem is solved by solving consecutively a convection problem and a diffusion problem. The convective problem is solved over the physical triangular mesh, by applying a recently proposed analytical procedure, where the unknown variables are located at the three vertices of each triangles. The diffusive problem is solved by adopting the flux discretization proposed in the companion paper (A novel procedure for the solution of heterogeneous anisotropic transport problems. Part 1: the diffusion problem) over a modified mesh. The mesh adjustment, based on a series of edge swaps, suggested in the companion paper for steady-state diffusion problem, attains the Delaunay condition for all the triangle sides without changing the original nodes location and also maintains the internal boundaries. In the adjusted mesh, the control volume relative to node i is defined by linking the "modified" circumcentres of the triangles sharing node i with the midpoints of triangle edges. The solution computed by the convection problem is discontinuous at triangle element interfaces, so that, at each time iteration, at the end of the solution of the convection problem, a single u value at each node i of the mesh is computed by preserving the mass for the triangles sharing the same node. This represents the initial state for the solution of the diffusive problem. Since diffusion tensor is time-dependent, the mesh adjustment procedure for the solution of the diffusive problem modifies the mesh at each time level, but no significant changes occur between two consecutively time iterations if the starting mesh for the diffusive problem is the one obtained at the previous iteration. The numerical methodology is tested with literature tests for transport tracers and underground waste disposal simulations and a comparison of the computed solution with the corresponding ones given by other reference models is presented too.