DISCONTINUOUS GALERKIN METHODS FOR VARIABLE DENSITY GROUNDWATER FLOW AND SOLUTE TRANSPORT

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Saltwater intrusion is the displacement of fresh water by saline water. Saltwater intruding into littoral zones and eventually into coastal aquifers is of significant interest to water managers. Coastal aquifers are generally part of a larger system where freshwater aquifers are hydraulically connected with a saline surface-water body. These aquifers are characterized by salinity variations in space and time, sharp freshwater/saltwater interfaces which can lead to dramatic density differences, and complex groundwater chemistry. Numerical simulation of these problems requires solving systems of coupled flow and transport equations with irregular geometry and heterogeneity which can require significant HPC resources. In this research, we develop a new DG formulation for the non-linear coupled flow and solute transport equations that model saltwater intrusion. The equations are coupled through a density equation of state. Since the transport equation can be advection-dominated, it presents numerical difficulties when trying to resolve the sharp moving fronts or freshwater/saltwater transition zones that characterize coastal aquifers. It is essential that the numerical method preserve the steep gradients with minimal oscillation and numerical diffusion. For the flow equation, a high numerical accuracy for the Darcy velocity is essential to capturing the correct physics of the problem. A poor approximation can produce spurious numerical velocities that lead to increased advective and dispersive transport of solute. DG methods have been proven to be well suited to model subsurface flow and transport problems and have many desirable characteristics over CG models in the areas of numerical stability, mesh and polynomial approximation adaptivity and the use of non-conforming meshes. Previously, DG methods have been utilized to discretize the transport equation coupled with mixed finite element and finite volume methods for the flow, but to our knowledge, a full DG model has not been formulated for the system. In our formulation, we discretize the flow equation using a Local DG (LDG) method which gives a locally conservative, consistent approximation of the Darcy velocity. For the transport equation, we utilize a non-symmetric, interior penalty Galerkin (NIPG) method for the diffusive flux with standard and Lesaint-Raviart upwinding methods for the conservative and non-conservative advective flux formulations. An implicit time discretization is used with a sequential coupling method and the discretized system is solved utilizing the DoD Proteus Toolkit software. We present the model formulation, initial numerical results for several existing 2D benchmark problems, initial stability and error estimates and show the compatibility of the flow and transport formulations.