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TOWARD REDUCTION OF UNCERTAINTY IN COMPLEX MULTI-RESERVOIR RIVER SYSTEMS

Arturo S. Leon[*,†,§], **Nathan L. Gibson**[*,‡] **and Christopher H. Gifford-Miears**[*,†]

[†]School of Civil and Construction Engineering
[‡]Department of Mathematics
[*]Oregon State University, Corvallis, OR 97331, USA
[§]e-mail: arturo.leon@oregonstate.edu, web page: `http://web.engr.oregonstate.edu/~leon`

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Summary. In this paper we present a novel framework for better modeling uncertainty in complex systems and demonstrate the approach applied to regulated river systems. In this framework, only the stream inflows are assumed to be purely stochastic. Other uncertain quantities are correlated to the uncertainty of the stream inflows using the dynamics of the river as the physical description of the system. We represent uncertainty in stream inflows via an error term modeled as a stochastic process. The resulting system is discretized (in random space) using stochastic collocation thus requiring only deteministic system solves. The dynamics of the river system are simulated efficiently using the performance graphs approach.

1 INTRODUCTION

Uncertainties play a major role in the operation of any system. Large ranges of uncertainty may lead to inadequate operation strategies and hence may not be useful for the operation of a system. Large uncertainties may result in flooding scenarios that in turn may cause loss of human lives and large economic losses. An inadequate operation strategy may also result in an unnecessary release of large volumes of water, which would be in conflict with the long-term objectives of the system, such as those of maximizing water storage for irrigation and hydropower production. In the case of regulated river systems, uncertainties come basically from two sources; (a) inflows to the river system (external sources) and (b) variables associated with the operation of the system (internal sources). Many methods that model uncertainties assume that they are all purely stochastic and independent, both spatially and temporally. Each stochastic variable is then represented by, for example, a probability distribution function (PDF). These methodologies may result in unnecessarily large uncertainties that may not be useful for the operation of a system.

In seeking to optimize a system, the natural question is how the system should be operated given the uncertainties affecting the system. In the case of regulated river systems with multiple reservoirs the main source of uncertainty for the operation of each reservoir is the stream inflows. Other uncertainties, such as reservoir outflows, water stages and gate positions are present as well. However, we argue that not all uncertainties should be treated as independent and purely stochastic. Some uncertainties can be correlated to those "assumed" to be purely stochastic either spatially, temporally or both using the physical description of the system. For instance, in complex reservoir systems, the uncertainties of outflows at each reservoir can be correlated to the uncertainties of the inflows to the system.

For a complex regulated river system, we propose to correlate the uncertainties using the dynamics of the river system. We choose to use the *dynamics of the river* as the physical description of the system because the operation of a regulated river system under extreme events may result in flooding scenarios that are highly dynamic. A flooding event may start from anywhere in the river system. It may start from upstream (e.g., large inflows), from downstream (i.e., high water levels at downstream) or laterally from the connecting reaches (e.g., water levels at river junctions are near the reach banks). Additionally, flooding events for a given system can demonstrate variability, depending on the inflows to the river system and the antecedent boundary conditions. Accounting for system flow dynamics is also important because the flow conveyance from one reservoir to another is not instantaneous but depends on the capacity of the connecting reaches, the capacity of the associated gates and outlet structures and the dynamic hydraulic gradients.

1.1 Review of methods for simulation of flow dynamics in rivers systems

In the following we present a discussion on unsteady flow routing (river system flow dynamics) and on state-of-the art methods for simulating flow dynamics in complex river systems. Due to space limitations this discussion is limited to one-dimensional models. In a one-dimensional context, unsteady flows in open-channels are typically represented by the Saint-Venant equations, a pair of one-dimensional partial differential equations representing conservation of mass and momentum for a control volume, which is shown in conservative differential form in Equations (1) and (2).

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\cos(\theta)\frac{dy}{dx} - g(S_0 - S_f) = 0 \tag{2}$$

In these equations, x = distance along the channel in the longitudinal direction; t = time; Q = discharge; A = cross-sectional area; y = flow depth normal to x; θ = angle between the longitudinal bed slope and a horizontal plane; g = acceleration of gravity; S_0 = bed slope and S_f = friction slope. Appropriate initial and boundary conditions are required to close the system. Due to the presence of non-linear terms in Equation (2), no closed-form solution exists, except for very special conditions. The equations are therefore often solved numerically. In a channel network involving numerous branches, the system of equations that must be solved becomes extremely large and the application of the full Saint-Venant equations becomes inefficient for real-time flood operation because of the significant computational requirements and error accumulations.There are essentially two options, which can be used independently or in conjunction with one another, for finding a more efficient solution to the equations. One option is to use different numerical techniques to solve the differential equations. The other option is to approximate the Saint-Venant equations by neglecting one or multiple terms from the full dynamic-wave equation.

Even though the dynamics of a river system can be simulated with very good accuracy, its actual implementation in river operation models is challenging mainly because of (a) lack of robustness and (b) computational burden of current unsteady flow models.Some of the existing routing models, especially those that are one-dimensional, are highly robust for a wide range of conditions but not necessarily for all. For instance the widely-known unsteady HEC-RAS model can provide accurate results for a large range of conditions but may fail completely for some others. It is worth mentioning that when the HEC-RAS model encounters problems with convergence, the simulation is not stopped but rather continued assuming pre-specified conditions (e.g., critical flow). Clearly, therefore, the results after the convergence problems should be treated with caution.

Although many schemes have been proposed to solve the Saint-Venant equations numerically, a common feature of these numerical solutions is that the accuracy and stability of the solutions are dependent on the temporal and spatial steps selected for routing. Small temporal steps are often needed for computational stability and small spatial steps are often needed to achieve high precision. The development of a hydraulic model that is computationally stable for large time steps would be advantageous in the routing of these flows.

Recently, a numerically efficient and robust approach for consolidating backwater profiles of a single channel reach into a family of delivery curves, expressed in the form of water surface elevations (stages) or water depths at the ends of the channel reach for different constant discharges was developed[1]. This approach was called the Hydraulic Performance Graph (HPG).

The HPG is unique to a channel reach with a given geometry and roughness, and can be computed decoupled from unsteady boundary conditions (BCs) under gradually varied flow (GVF) scenarios. HPGs only need to be revised (updated) when geomorphic changes modify the geometry or roughness characteristics of the channel. It is worth mentioning that any one-dimensional (1D), two-dimensional (2D) or three-dimensional (3D) **gradually varied flow model** can be used for generating HPGs. A significant advantage of the HPG approach with respect to other routing models is that any error attained during the pre-computation of the hydraulics (e.g., due to instability) can be detected and therefore corrected before the routing process (e.g., recompute simulations with different discretization parameters).

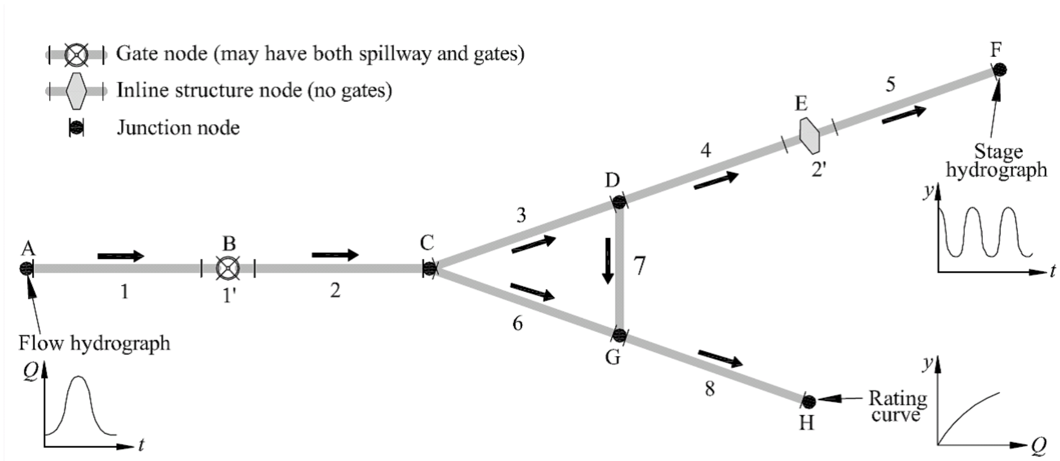

Figure 1: Schematic of a simple network system[2]

Even though HPGs and VPGs result in highly robust and numerically efficient models for hydraulic routing, previous models based on the HPG/VPG approach were not formulated for a general river network. Very recently[2] the HPG/VPG approach was expanded to complex river networks. A model was formulated for simulating dentritic and looped networks with successful results.

2 Incorporating Uncertainty

While the physical processes of most complex systems may be well understood they are typically not fully exploited for reducing levels of uncertainty in the operation of these systems. The proposed uncertainty framework aims to combine "purely stochastic processes" (not well understood or unpredicted at this stage) with the physical description of the system (e.g., flow dynamics in the case of complex regulated river systems) with the goal of better modeling uncertainties and hence reduce the ranges of the confidence intervals.

We will consider the sample network system presented in Figure 1. The (nonlinear) relationship between variables $\vec{X} = [y_{d_1}, \ldots, Q_{d_8}]$ (water stages y and flow discharges Q upstream and downstream of each river reach) on each timestep has been given by Leon et al.[2]. The flow dynamics are simulated using the HPG/VPG approach for a river network implemented in OSU Rivers, a state-of-the-art, multi-objective, simulation-optimization model that is intended for real and non-real time operation of complex river systems. To take advantage of this software which is under continued development, we incorporate the uncertainty framework *non-intrusively* into OSU Rivers.

For the efficient computation of the uncertainty components, rather than doing sampling of the input distributions, we propose to explicitly model the random space (via random variables and processes) and perform a generalized Polynomial Chaos (gPC) representation[4].

This approach uses an orthogonal polynomial expansion in random space to represent the stochastic input quantities as well as the solution to the system. For example, the quantities may be represented by a Karhunen-Loeve expansion if the randomness is time-dependent. Convergence of polynomial chaos methods can be shown to be exponential in the number of basis functions. In order to compute the coefficients of the expansion of the solution, integrals (resulting from a Galerkin projection) must be evaluated. Due to the polynomial representation, these integrals may be computed exactly, however this approach in general leads to a large coupled system of equations and this new system must be discretized in space and time (e.g., an *intrusive method* which changes the system to be solved).

Alternatively, one may apply Gaussian quadrature to the integrals thus requiring only that the original system to be solved at specific points in random space. This approach is called Stochastic Collocation[4] and is distinct from sampling methods in that the Gaussian nodes are pre-determined. By recycling existing deterministic codes, this *non-intrusive* approach is as flexible as traditional Monte-Carlo methods (while being more efficient with respect to convergence rates). In particular, the system evaluations may be done independently, *in parallel*. Furthermore, should the distribution of inputs change (e.g., updated predictions), previous system evaluations can be reused by simply updating the Gaussian weights.

3 Framework Description

To illustrate the idea of our approach, we present the following example. Consider the quantity solution quantity Q_{u_1} representing flow discharges upstream of reach 1 in Figure 1. We may assume that a prediction of this quantity $\overline{Q}_{u_1}(t)$ is given, but that an uncertainty envelope may be applied, resulting in $Q_{u_1} = Z\overline{Q}_{u_1}$. We may assume that the uncertainty around the prediction $\overline{Q}_{u_1}(t)$ is *relative*, for example, with magnitude 10%. If we introduce the standard random variable $\xi \sim Beta(\alpha, \beta)$ with support $[-1, 1]$, then

$$Q_{u_1} = \overline{Q}_{u_1} + 0.1\xi\overline{Q}_{u_1}. \tag{3}$$

If $\alpha = \beta$, the distribution F is a symmetric Beta centered around mean 0; for the special case $\alpha = \beta = 0$ this is simply a uniform distribution. Equation (3) represents a *polynomial chaos expansion* of the random input (in general, an expansion only represents an approximation, however in this case it is exact). In a Galerkin method we seek to determine the coefficients of a gPC expansion of each component of the solution vector $\{y_{d_1}, \ldots, Q_{d_8}\}$, or some function of the solution vector. To do this, we may take a Galerkin projection of the original system with these expansions substituted in for the solution quantities.

For example, consider the most downstream reach, Q_{d_8}. Its representation in terms of a degree P expansion

$$Q_{d_8}^P(t, \xi) = \sum_{i=0}^{P} v_i(t)\phi_i(\xi)$$

where $\phi_i(\xi)$ are the basis functions (Jacobi polynomials in the case of a Beta distribution of inputs). Each gPC expansion coefficient is given by

$$v_i(t) = \mathbb{E}[Q_{d_8}(t,\xi)\phi_i(\xi)] = \int Q_{d_8}(t,\xi)\phi_i(\xi)dF(\xi) \tag{4}$$

i.e., the expected value with respect to F of the *true solution* times the (normalized) basis function. The computation of (4) can be performed efficiently via Gaussian quadrature

$$\mathbb{E}[Q_{d_8}(t,\xi)\phi_i(\xi)] \approx \sum_{j=1}^{N} w_j Q_{d_8}(t,\xi_j)\phi_i(\xi_j).$$

The nodes ξ_j of the quadrature rule are the roots of a determined orthogonal polynomial in the support of F and the method has the highest degree of precision possible. Stochastic Collocation thus requires only deterministic system solutions for the fixed values $\{\xi_j\}_{j=1}^N$ of the random variable ξ. After simulations are performed, the gPC expansion for any function of output (non-linear, non-smooth or even discontinuous) may be easily constructed as follows

$$v_i(t) = \mathbb{E}[f(\vec{X}(t,\xi))\phi_i(\xi)] \approx \sum_{j=1}^{N} w_j f(\vec{X}(t,\xi_j))\phi_i(\xi_j).$$

In practice, only desired functions of the solution of the system need be represented explicitly. For instance, in a multi-objective optimal control framework, the flood volumes and hydro-power generations may be the actual quantities of interest. The above example can be restated via a mapping from the solution quantities to the desired quantities. It is important to note that this mapping need not be linear, nor need it even be continuous, however in the latter case exponential convergence is sacrificed in favor of algebraic. For example, we may wish to use Flooding Performance Graphs as a nonlinear and continuous (but not continuously differentiable) mapping to flood volumes.

4 Numerical Example

In Figure 2 we present preliminary results of the propagation of uncertainty via the proposed approach. We computed the ninth degree gPC expansion coefficients of the flood volume (FV) in a single reach simulation given a relative uncertainty envelope on the input flow with magnitude 10%, as described above in (3), using $\xi \sim Beta(10, 10)$. We assumed that the inflow was given by $\overline{Q}_{u_1}(t) = A\exp(-bt)\sin^2(\omega t)$ for some realistic values of b, ω, and amplitude A. The expansion is clearly nonlinear yet low degree suggesting only a few polynomial basis functions are necessary for accurate representation. We note that the flood volumes are computed efficiently using the Flooding Performance Graph (FPG) approach. The actual expansions of the solution variables need not be explicitly computed.

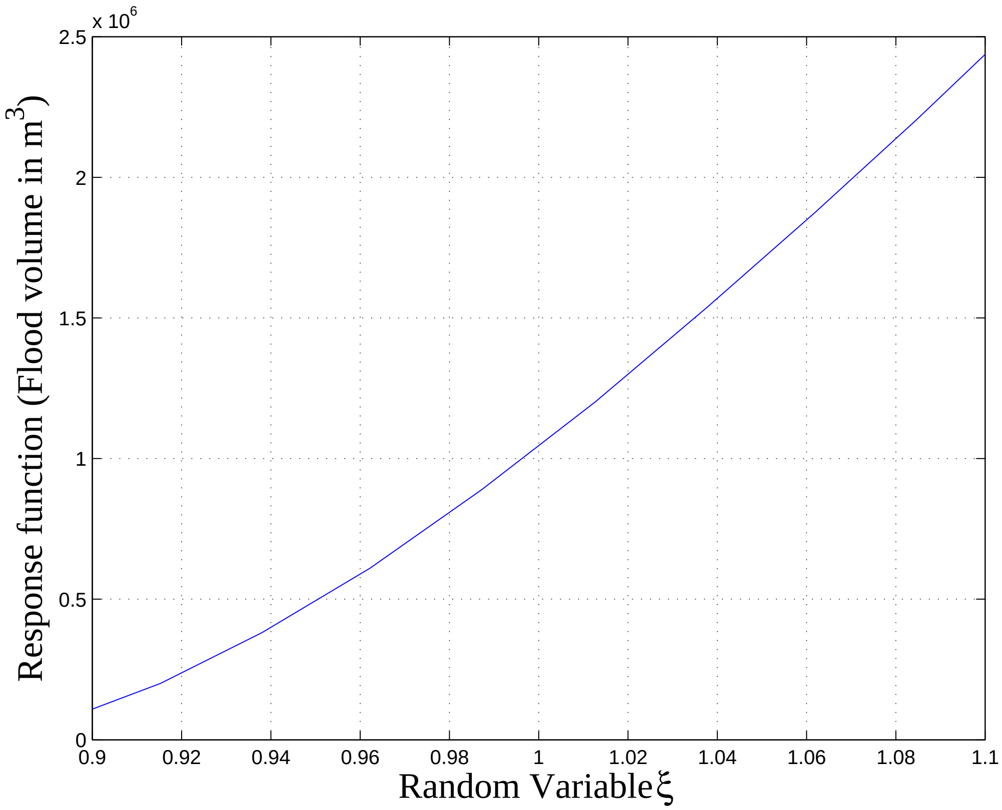

Figure 2: The gPC expansion (*response function*) for the flood volume in a single reach simulation using predictions of Q_{u_1} with uncertainty modeled by a Beta distribution

The output of the system Q_{d_1} in a single reach is plotted in Figure 3 as a function of time, along with the single standard deviation envelope. We remark that this envelope is not simply a relative error, but instead indicates more initial uncertainty than would be otherwise expected, and less uncertainty at the trough than a relative error formulation would predict. In this example, a Beta distribution of inflows was again assumed $\xi \sim Beta(10, 10)$.

5 Conclusions and Future Work

In this paper we presented a novel framework for better modeling uncertainty in complex systems and demonstrate the approach applied to regulated river systems. For the quantification of uncertainty, rather than doing sampling of the input distributions, we explicitly model the random space. The resulting system is discretized (in random space) using stochastic collocation. Upon computation of the expansion coefficients for the quantities of interest, we essentially have an analytical representation of the stochastic solution in polynomial form. This allows, among other things, various solution statistics to be easily obtained.

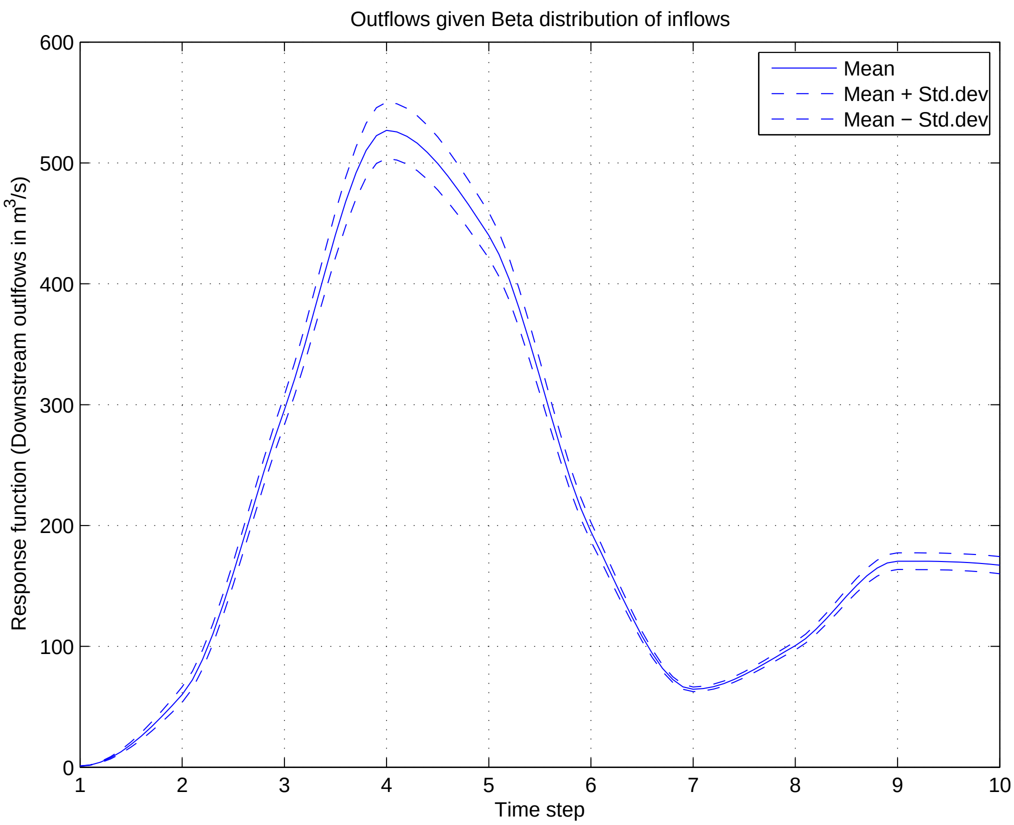

Figure 3: The output of the system Q_{d_1} in a single reach along with plus and minus one standard deviation

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