

# APPLICATION OF THE TRUNCATED PLURIGAUSSIAN METHOD TO DELINEATE HYDROFACIES DISTRIBUTION IN HETEROGENEOUS AQUIFERS

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**Summary:** We analyze the performance of the Truncated Plurigaussian Simulation method (TPS) for the reconstruction of the internal hydrofacies architecture within large scale aquifer systems. This allows including conceptual geological information in a characterization procedure. The impact of adopting a TPS or a traditional indicator-based approach to generate multiple realizations of geo-materials distributions is assessed through numerical Monte Carlo analysis of vertically averaged hydraulic heads collected during a simulated pumping test.

## 1 INTRODUCTION

Characterizing aquifers heterogeneity is a major issue in groundwater modeling. Geological heterogeneity of sedimentary bodies can be represented and modeled from depositional facies distributions. Statistical grid-based sedimentary facies reconstruction and modeling methods (FRM methods) can be employed to provide accurate representations (reconstructions or models) of facies distribution at a variety of scales. These can be conditioned to hard and soft data and enable to quantify the effects of geological heterogeneity. Falivene et al. 2007<sup>1</sup> describes a workflow and provides an overview, classification, description and illustration of the most widely used FRM methods (deterministic and stochastic) which include the pixel-based methods termed as sequential indicator (SISIM) and truncated plurigaussian (TPS) simulation.

Litho-facies simulations based on the concept of indicators are widely used in the petroleum industry. The truncated Gaussian simulations (TGS) method is one of the few available models that guarantees consistency between the variogram and cross-variogram and enables conditioning simulations. The use of several Gaussian functions to define hydro-facies extends the potential of the TGS method and is the cornerstone of the truncated plurigaussian method. The principles of this simulation technique are described in Galli et al. 1994<sup>2</sup>. TPS allows taking into account complex transitions between material types and simulating anisotropic distributions of litho-types while the TGS explicitly allows only sequentially ranked categories.

TPS is typically employed to simulate geological domains (facies) in petroleum reservoirs and mineral deposits, hydrofacies in aquifers, or soil types at a catchment scale with the aim of (a) assessing the uncertainty in the internal boundaries demarcating geo-materials within the domain and (b) improving the geological constraints in the characterization of quantitative attributes (Emery 2007<sup>3</sup>). Xu et al. 2006<sup>4</sup> and Emery 2007<sup>3</sup> develop simulation software implementing and making accessible the TPS method. Mariethoz et al. 2011<sup>5</sup> illustrate an application of TPS to assess pollutant migration in highly heterogeneous media in an aquifer located in Switzerland.

On the other hand, sequential indicator algorithms are widespread geostatistical simulation techniques that rely on indicator (co)kriging and are applied to a wide range of datasets. Emery 2004<sup>6</sup> highlights limitations of SISIM upon examining the conditions under which a set of realizations (*i*) is consistent with the input parameters and (*ii*) honor the model parameters. Comparisons between different geostatistical models have been published in the literature, e.g. Dell’Arciprete et al. 2012<sup>7</sup> (and references therein).

Our goal is to examine the effect of representing geologic heterogeneity through truncated plurigaussian simulations (TPS) on the hydraulic response of an aquifer subject to a pumping stress. The analysis is performed by way of a numerical example which makes use of geological information collected in a groundwater system located in the Adda-Serio rivers basin (Lombardia, Italy). An ensemble of hydrofacies distributions is reconstructed on the basis of sedimentological data upon employing (a) an indicator-based approach (SISIM) and (b) the TPS method. While the former methodology only relies on a variogram-based analysis, the latter allows integrating geological information related to different sources in the simulation procedure. TPS allows employing effectively both local/regional and conceptual geological information to infer the distributions of hydrofacies and, therefore, the associated hydraulic parameters. A pumping test is then simulated within the generated (Monte Carlo) sets of (conditional) three-dimensional hydrofacies distributions. The observed collection of vertically-averaged drawdown responses calculated at a set of observation piezometers allows comparing the effect of the stochastic simulation scheme on the response of the system.

## 2 DATA-BASE AND AREA UNDER STUDY

The data-base adopted for the simulations of the facies distribution in our computational example is associated with a groundwater system located between the Adda and Serio Rivers across the Provinces of Cremona and Bergamo (Lombardia, Italy), where a springs’ network is still active. The aquifer is exploited mainly for agricultural purposes. Significant lowering of the

water table due to pumping causes major environmental concerns related to the vulnerability of the springs in the northern part of the study area, with a strong impact on the environmental and ecological system.

Several surveys have been performed on the area (see, e.g., Bianchi Janetti et al. 2011<sup>8</sup>, Vassena et al. 2012<sup>9</sup> and references therein) and provide detailed sedimentological information. These include well locations and exhaustive lithological descriptions. Hard data employed in our geostatistical simulation correspond to a dense (approximately 1 borehole/2.3 km<sup>2</sup>) boreholes network. Figure 1 depicts a sketch of the area located in the Lombardia region where the data-set has been collected. Data re-classification is performed to obtain a simplified representation of the complex and heterogeneous architecture that is inherent to the aquifer system. The different hydrofacies identified in the region are described in terms of categorical variables, or indicators, and represent the following categories: fine materials (F1), sand (F2), gravel (F3), compact conglomerates (F4) and fractured conglomerates (F5). Their associated hydraulic conductivity values are taken as  $K_1 = 1.12 \cdot 10^{-7}$  m/s,  $K_2 = 1.79 \cdot 10^{-5}$  m/s,  $K_3 = 7.16 \cdot 10^{-4}$  m/s,  $K_4 = 9.09 \cdot 10^{-5}$  m/s,  $K_5 = 6.05 \cdot 10^{-3}$  m/s. These values have been taken from previous studies (see e.g. Bianchi Janetti et al. 2011<sup>8</sup>) performed close to the target area and are considered as uniformly distributed within each hydrofacies class.

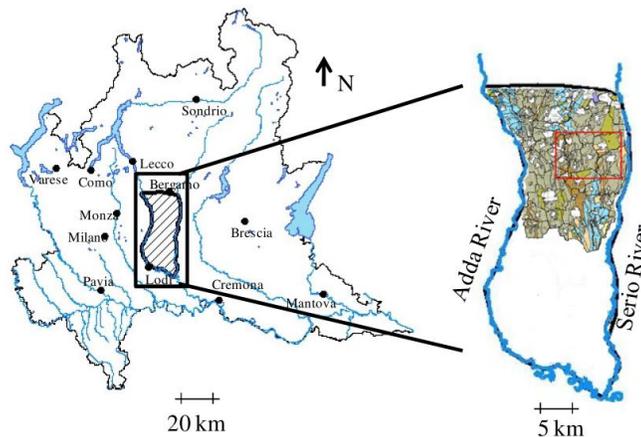


Figure 1: Sketch of the area located in the Lombardia region, Italy, where the borehole network is located. The red square indicates the domain where the geostatistical analysis of sedimentological data has been performed.

### 3 SIMULATIONS WORKFLOW

#### 3.1 Variogram analysis

A detailed variogram analysis of the indicator-based variables is performed. Three principal anisotropy directions are identified (North-South, N-S; West-East, W-E; vertical). This finding is consistent with field observations and lithological maps from the area under study. Table 1 summarizes the key characteristics of the variogram models adopted to interpret the spatial variability of geomaterials within the system. The values of the fourth column (SISIM) represent

parameters assigned to the facies indicator variograms, whereas the values in the last column (TPS) represent the parameters assigned to the two gaussian auxiliary variables. As an example, Figure 2 reports the sample variogram for facies F1 along the N-S and vertical directions. The interpretive models selected are also reported.

	Variogram model	Sill	Ranges N-S / W-E / Vertical [m] – SISIM	Ranges N-S / W-E / Vertical [m] – TPS
F1	Exponential	0.246	1830 / 1280 / 37	900 / 600 / 19
F2	Exponential	0.017	700 / 470 / 20	900 / 600 / 36
F3	Exponential	0.173	1240 / 910 / 34	900 / 600 / 36
F4	Exponential	0.167	860 / 670 / 20	900 / 600 / 36
F5	Exponential	0.098	1220 / 900 / 36	900 / 600 / 36

Table 1: Variogram models adopted in the hydrofacies analysis and their associated parameters. Nugget is equal to 0, for all models.

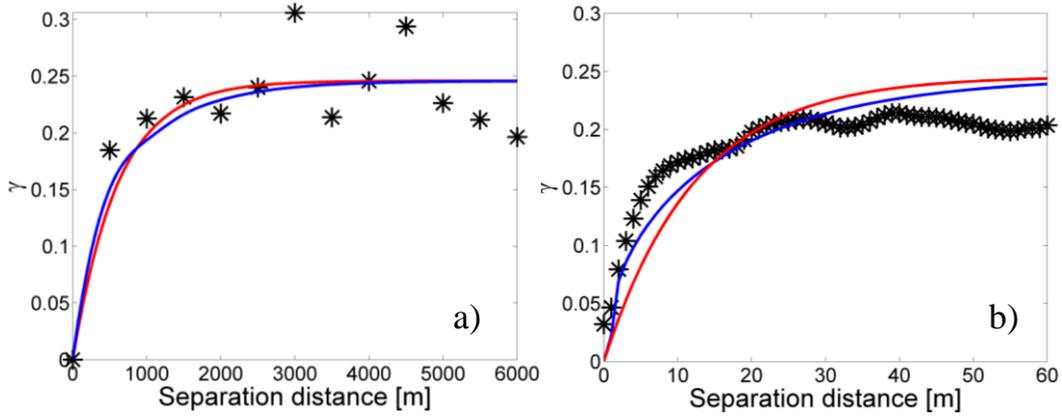


Figure 2: Experimental (black symbols) and modeled (SISIM in red and TPS in blue) variograms for facies F1 along the (a) N-S and (b) vertical directions.

### 3.2 Numerical simulations

A number  $N = 500$  of three-dimensional realizations of facies distributions are generated on the basis of the TPS and SISIM schemes. The simulation grid comprises  $74 \times 86 \times 51$  elements or nodes (respectively organized into rows along the N-S direction, columns and layers). The grid spacing is set to 50 m in the horizontal plane and to 2 m along the vertical direction, resulting in a domain with a planar extension of  $3.7 \times 4.3 \text{ km}^2$  and a thickness of 100 m.

Table 2 lists the materials volumetric proportions inferred from the available data and employed in the simulation process. It also includes the average ( $\mu$ ) and standard deviation ( $\sigma$ ) of the volumetric proportions calculated for each facies via TPS and SISIM. Figure 3 illustrates the rate of convergence of the mean and standard deviation of the volumetric proportion of facies F1 and F4 through the Monte Carlo process. It is noted that sample averages of volumetric proportions appear to converge slightly faster for SISIM- than for TPS-based simulations. For

both generation schemes, 500 iterations provide stable first and second moments of volumetric proportions of geomaterials. The results of Table 2 indicate that: (i) the average coefficient of variation associated with the generated facies proportions is 0.10 for SISIM and 0.42 for TPS; and (ii) the average standard deviation associated with the sample mean proportions (calculated as  $\sigma / \sqrt{N}$ ) is 0.06 and 0.22, for SISIM and TPS simulations, respectively. As a result, the input volumetric proportions of facies F2 – F5 are comprised within intervals of width equal to  $\pm 3\sigma / \sqrt{N}$  around the corresponding sample mean for the TPS procedure.

	Input proportions	$\mu$ (SISIM)	$\sigma$ (SISIM)	$\mu$ (TPS)	$\sigma$ (TPS)
F1	43.6%	39.7	1.8	46.3	8.8
F2	1.8%	2.2	0.5	1.6	1.5
F3	22.3%	22.5	1.5	22.0	6.1
F4	21.2%	23.7	1.4	20.5	4.3
F5	11.1%	11.9	1.3	9.6	4.6

Table 2: Input and simulated volumetric proportions for the five hydrofacies. Average ( $\mu$ ) and standard deviation ( $\sigma$ ) of the volumetric proportions are calculated for each facies on the basis of  $N = 500$  realizations.

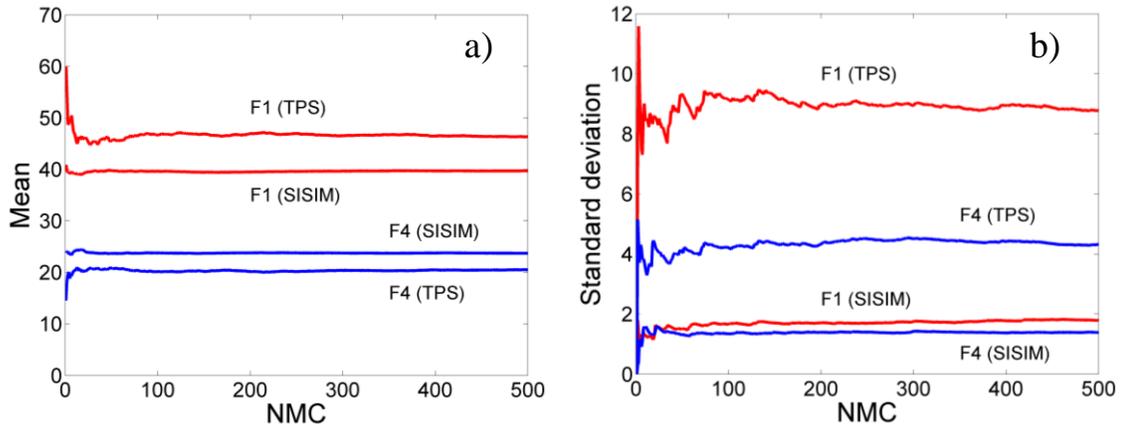


Figure 3: Dependence on the number of Monte Carlo iterations, NMC, of the (a) mean and (b) standard deviation of the generated volumetric proportions for hydrofacies F1 (red) and F4 (blue) with the two generation methods analyzed.

Following Dell’Arciprete et al. 2012<sup>8</sup>, the observed inability of the two methods to represent the input volumetric proportions may be explained due to the low ratio (<2.3%) between the conditioning data nodes (around 7,500) and the high amount of geostatistically simulated cells (approximately 325,000).

As an example, Figure 4 reports two selected simulations obtained via SISIM and TPS. It can be noted that TPS renders an improved continuity of geological structures. These are elongated along the N-S direction, consistently with observations from surface lithological maps in the area under study. Both methods reproduce the anisotropy patterns detected in the system.

The numerical mesh adopted for the flow simulations is built by further refinement of the hydrofacies generation grid, resulting in a total of 148 rows along the N-S direction, 172 columns along the W-E direction and 51 layers along the vertical direction.

The generated Monte Carlo sets of (conditional) three-dimensional hydrofacies distributions are employed to simulate a convergent flow scenario due to a pumping test. The observed ensemble of drawdown responses calculated at a set of observation piezometers allows comparing the effect of the stochastic simulation scheme on the response of the system. A fully penetrating pumping well is located approximately at the center of the domain. A fixed pumping rate of 3,000 m<sup>3</sup>/day is assigned to the well nodes proportionally to the hydraulic conductivity of the model blocks. Constant heads of 172.5 m and 145 m are imposed on the Northern and Southern edge, respectively, in agreement with typical average values of observed regional hydraulic gradients. No-flow conditions are imposed along the Eastern, Western and bottom boundaries. Flow simulations are performed under steady state in the framework of the widely used and tested software Modflow (McDonald et al. 1988<sup>10</sup>).

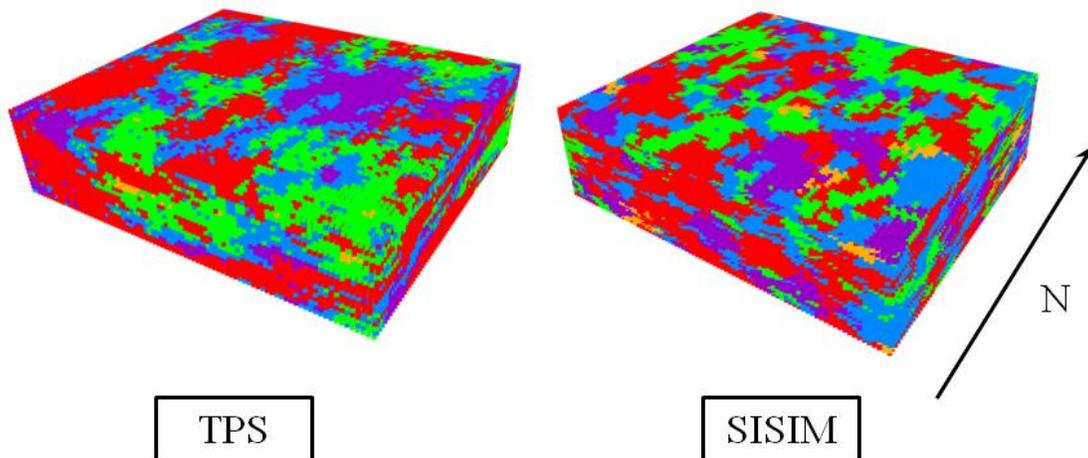


Figure 4: Two realizations of the target area, obtained by means of TPS and SISIM. Color scale: red (F1); orange (F2); green (F3); blue (F4); violet (F5).

The hydraulic head obtained from each flow simulation and for each geostatistical realization is averaged along the vertical, at locations placed along two orthogonal (N-S and W-E) sections centered at the pumping well. This allows simulating head readings that are typically representative of a state of the system which is integrated over the whole screened intervals of the observation boreholes. Heads are also averaged according to 25-m vertical segments to simulate the effect of partial penetration/screening of the observation boreholes.

Figure 5 illustrates the distributions of the ensemble mean water heads averaged along the thickness of the simulation domain for both simulation methods. Figure 6 reports corresponding results obtained upon vertical-averaging of hydraulic heads along the upper and bottom 25-m screened segment. Intervals of width corresponding to a standard deviation are reported around

the ensemble means to provide an indication of the degree of variability induced by the generation method throughout the ensemble of realizations.

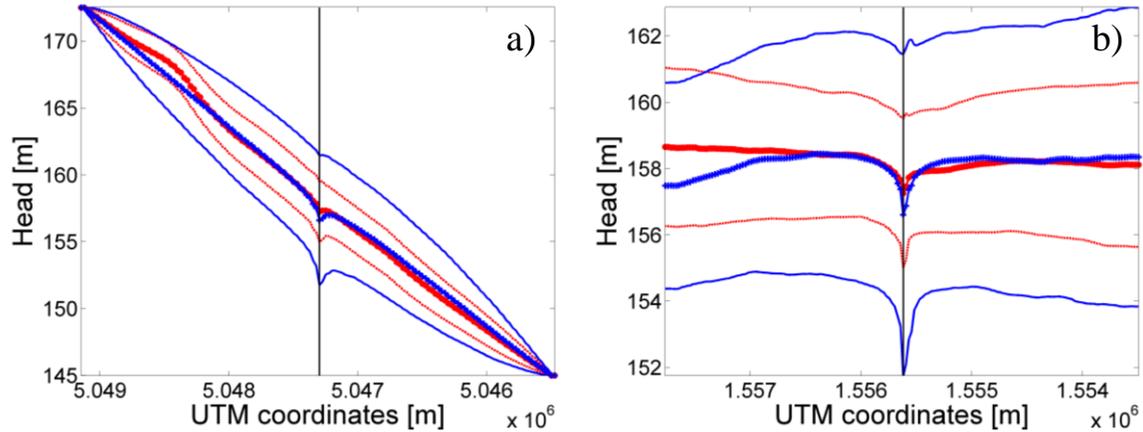


Figure 5: Distribution along the (a) N-S and (b) W-E direction of ensemble averages of hydraulic heads averaged along the thickness of the domain and calculated through SISIM (red symbols) and TPS (blue symbols). Intervals of width corresponding to a standard deviation are reported around the ensemble means.

The average behavior stemming from the application of the two methods is quite similar along both directions: the head values are highest for SISIM in the northern and western sub-domains and for TPS in the southern half. The largest drawdown at the well is obtained through the TPS method (Figure 5). TPS renders the largest ensemble standard deviation of hydraulic heads, consistently with the results illustrated in Table 2.

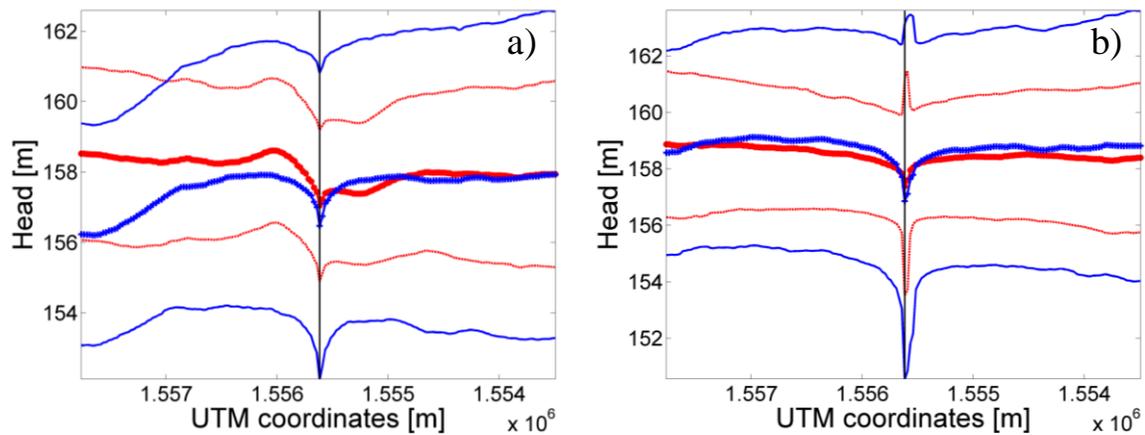


Figure 6: Distribution along the W-E direction of ensemble averages of hydraulic heads after vertical-averaging along (a) the first 25 m or (b) the bottom 25 m of the system and calculated through SISIM (red symbols) and TPS (blue symbols). Intervals of width corresponding to a standard deviation are reported around the ensemble means.

## 4 CONCLUSIONS

Our numerical Monte Carlo simulations suggest that setting geological contact rules, as considered within a TPS simulation scheme, can lead to an increased variability in the internal architecture of sedimentological facies distributions. As a result, TPS-based flow simulations render enhanced variability of system responses (e.g., hydraulic heads) due to the effect of pumping stresses.

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